Quadratics nice problems

October 20, 2026

This is a collection of problems I have found to be accessible to GCSE and A-Level students, while giving them a feel for problem-solving and letting them play around and really twirl in their hands the properties of quadratic equations and the discriminant.

1. Suppose $y = ax^2 + bx + c$ has no roots, and $a + b + c > 0$. Find the sign of c.

2. True or False: if $b > a + c > 0$, then $ax^2 + bx + c = 0$ has two different roots.

   Is the condition $a + c > 0$ necessary?

3. Let $f(x)$ be a quadratic. Suppose $f(x) = 2 - 2x$ and $f(x) = x - 1$ both have unique solutions. Prove that $f(x) = 0$ has no solutions.

4. Take a quadratic equation with parameter a: $ax^2+(a+1)x+2a-1 = 0$. For which values of a does this equation have exactly one root?

These problems use Vieta's theorem. This is an extension beyond the default western school curriculum, yet is quite present in post-soviet mathematical education.

1. Take $x^2 + px + q = 0$. We know that three out of four values of x_1, x_2, p, q belong to the set $\{1, 2, -6\}$, what's the fourth value?

2. Take $x^2 + px + q = 0$. When are $2p$ and $p + q$ two different roots?

*3. A line intersects the graph of $y = x^2$ at points x_1 and x_2, and the x-axis at point x_3. Prove that $\frac{1}{x_1} + \frac{1}{x_2} = \frac{1}{x_3}$